Peak Current and Magnetic Flux Density Variations with Strip Width in Superconducting Microstrip Lines

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Abstract—**A computer model based on London's equations and Maxwell equations is used to investigate the characteristics of high T_c superconducting microstrip lines. Distributions of superconducting current densities inside the strip, the magnetic flux density, and the quality factor variations with the strip width and operating frequency are presented. The obtained results are very useful for CAD. It is observed that an empirical relation between the strip width and the peak current density on the strip can be deduced. The structure can be optimized to produce the highest quality factor or the largest current carrying capacity according to the application.**

I. Introduction

Obtaining the current density distributions on high T_c superconductor (HTS) transmission lines is a crucial step in designing and optimizing microwave planar circuits exploiting the interesting characteristics of HTS's. It is desirable if this step is primarily achieved using computer codes and CAD tools. The current carrying capacity of an HTS transmission line can be estimated knowing the current distributions inside the line and the critical current density and critical magnetic flux density of the HTS material used. Several theoretical approaches have been developed for analyzing microwave structures employing HTS materials (e.g., [1], [2]). However, the results obtained in these investigations are primarily the propagation characteristics of the structure. No information about the power handling capability was given. Winters *et al.* discussed the power carrying capability of superconductor rectangular waveguides [3]. Sheen *et al.* analyzed the current distributions in superconducting strip lines [4]. El-Ghazaly developed and implemented an approach for analyzing HTS microstrip lines, which is capable of obtaining current and field distributions in superconducting microwave planar transmission lines [5]. This approach is very flexible, and can incorporate complex issues such as the parameters nonlinearity and the anisotropy of HTS materials.

In this letter, the effect of strip width on current distributions, edge singularity and quality factor (Q) will be presented. Empirical expressions for the maximum current density at the edges, the maximum magnetic flux density, and the current carrying capacity of the superconducting strips will be derived. Such expressions are very useful for development of CAD tools.

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II. Effect of the Strip Width

A rigorous formulation based on Maxwell's equations, London's equations and the two fluid model is used to obtain the electromagnetic fields and the superconductor current density distributions. The derived equations are solved using a non-uniform finite-difference scheme. The details of this approach are described elsewhere [5]. The objective of this letter is to characterize the current and magnetic flux density distributions in the superconductor microstrip line.

This approach was applied to the microstrip structure shown in Fig. 1. Lossless dielectric substrate of thickness $d = 425$ μm with a relative dielectric constant $\epsilon_r = 23$ is used. Owing to the symmetry, only one half is analyzed and shown in Fig. 1. The superconducting strip has a thickness $t = 1$ μm, and the material is characterized by $T_c = 100$ K, the penetration depth at $T = 0$ K, is $\lambda(0) = 0.18$ μm, the density of electrons is 10^{21} cm^{-3}, and the conductivity of normal electrons $\sigma_n \approx 10^4$ S/cm at T_c. The ambient temperature is taken as 77 K.

A. Current Distributions

The superconducting current density distributions as functions of x, on the lower surface of the superconducting strip (i.e., $y = 0$) are shown in Fig. 2. Several cases of different strip widths (W) are shown. All of them carry a total current of 100 mA. For structures with relatively wide strips, the current density is fairly uniform over most of the strip. The current increases very rapidly near the strip edge due to the field singularity. The graph demonstrates that the ratio between the peak current density at the edge to the current density at the middle of the strip decreases as W decreases. This means that the current density becomes more uniform

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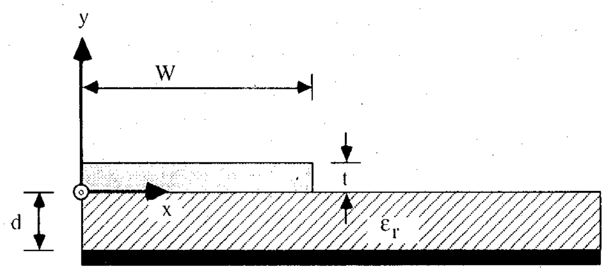

Fig. 1. Analyzed microstrip line. Only one half of the structure is shown.

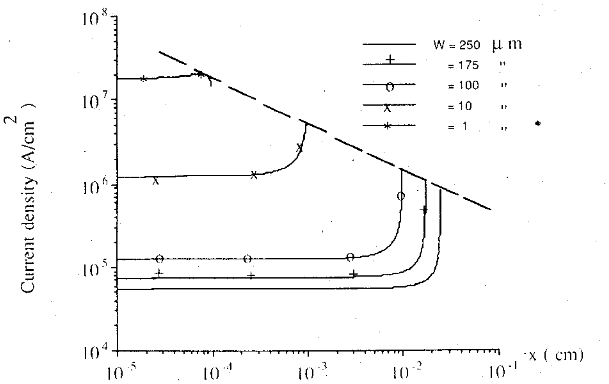

Fig. 2. Current density distributions at plane $y = 0$ for different strip widths.

for smaller W. However, the current density magnitude in the middle of the strip increases as W decreases since a smaller cross-sectional area becomes available. For strips of $W = 1$ μm, edge singularity vanishes completely. Obviously, the latter case is very similar to a wire over a ground plane, over which the current is almost uniform. One should notice that depending on the superconductor material used, the superconducting characteristics may be lost at small W due to the high current density, which may exceed the critical current density. In other words, strips of low W may not be able to carry the specified 100 mA, and as the strip width is reduced the total current has to be reduced accordingly.

Fig. 2 reveals an interesting relation between the peak current density and W. It is shown that the logarithm of the peak current density varies almost linearly with logarithm of W. An empirical relation can be derived, which is very useful for CAD. Taking this structure as a case study, this relation becomes

$$J_{\text{peak}} = 1.202 \times 10^5 e^{-0.53758 \ln(W)}, \qquad \text{A/cm}^2, \quad (1)$$

where W is in cm. This equation is derived with a total current of 100 mA. The maximum error resulting from using (1), relative to the numerical calculations, is about 5%. The actual current carrying capacity can be deduced from (1) thanks to linearity of the model. Given the material critical current density J_c (in A/cm^2), the maximum current that can be carried by the shown structure is given by

$$I_{\max} = 8.3187 \times 10^{-4} J_c e^{0.53758 \ln(W)}, \qquad \text{mA}. \quad (2)$$

B. Magnetic Flux Density Distributions

The magnetic flux density $\boldsymbol{B}$ is another important factor, which must be considered in the design of HTS microwave structures. To avoid losing the superconducting characteristics of the material, the magnetic flux density inside the HTS strip should not exceed the critical value B_c. Therefore to complete the design procedure, $\boldsymbol{B}$ distributions must be obtained as well. Fig. 3 illustrates the magnitude of $\boldsymbol{B}$ distributions for the cases depicted in Fig. 2 at the same total current of 100 mA. These distributions obtained at $y = 0$, which represents the surface of the highest magnetic field. Once again, B shows a strong singularity at the strip edges. It is also observed that the logarithm of the peak B varies almost linearly with logarithm of W on the logarithmic scale of Fig. 3. This enables the derivation of an empirical relation between the strip width and B_{peak} as follows.

$$B_{\text{peak}} = 3.254 e^{-0.60046 \ln(W)}, \qquad \text{gauss}, \quad (3)$$

where W is in cm. An expression estimates the current carrying capacity as a function of the strip width and B (in gauss), similar to (2), is also developed as follows.

$$I_{\max} = 30.731 B_C e^{0.60046 \ln(W)}, \qquad \text{mA}. \quad (4)$$

In designing HTS transmission lines, both (2) and (4) must be used simultaneously. The smallest of the resulting two currents is the correct line capacity.

Obviously, comprehensive study is required to include the dependence of these expressions on the other parameters including on the strip thickness, the substrate thickness, and dielectric constant. The aim of this letter is to report the calculated results, and to indicate that the derivation of empirical relations is possible based on an accurate numerical analysis as in [5].

C. The Quality Factor

The loss characteristics of this structure is represented by the Q calculations. Q is defined as

$$Q = \frac{\omega(W_c + W_m)}{P_1}, \quad (5)$$

where W_e and W_m are the time-average energy stored in the electric and magnetic fields per unit length of the line respectively, and P_l is the time-average power dissipated in the superconducting strip. The variations of the Q with the strip width, at different frequencies, are shown in Fig. 4. It is interesting to notice that the Q is very high at very small strip widths. This is because the current density distributions become more uniform, in the x-direction, at small W as shown in Fig. 2. As W is increased, the Q decreases due to the increased importance of the edge singularity. For relatively large W (i.e., $W > 200$ μm) the Q saturates. The latter phenomenon is understood by knowing that increasing W eventually leads to a more uniform current near the middle of the strip; and the current singularities at the edges do not change significantly. Of course, the Q decreases at high frequencies, as shown in Fig. 4, due to the higher electric field inside the superconductor that leads to higher losses. It is worth mentioning that the high Q at small W may drastically decrease due to the dielectric losses in the substrates.

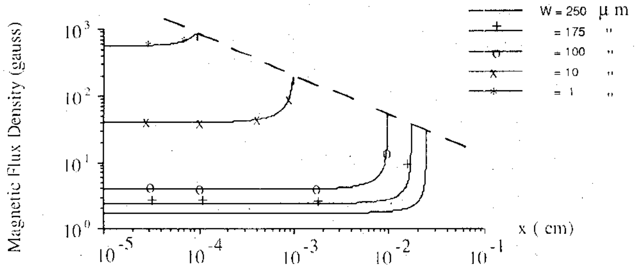

Fig. 3. Magnitude of the magnetic flux density distributions at plane $y = 0$ for different strip widths.

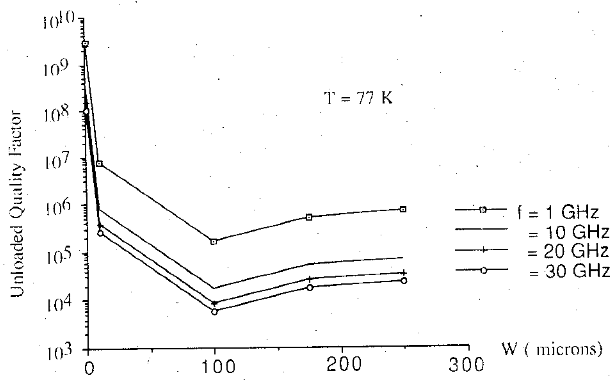

Fig. 4. Unloaded Q factor variations with the strip width as functions of frequency.

The presented results are very useful in designing HTS devices. For example, a small strip width may be used when a high Q structure is needed for an application that does not involve a high power transmission. On the other hand, to design a transmission line which carries large currents, the obvious choice is to use large W to increase the cross-sectional area. It is useful to know that increasing W beyond certain value, which is about 200 μm in this case, does not degrade the Q when all other parameters are kept constant.

III. Conclusion

The effect of the strip width on the current density and magnetic flux density distributions in HTS microstrip line is investigated. It is found that the edge singularities for both the current and magnetic flux densities become less important as the strip width is decreased. Empirical relations for the current carrying capacity in terms of the critical current density and the critical magnetic flux density are presented. Results showing the Q variations with the strip width are presented.

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